Weight-Induced Consumed Endurance (WICE): A Model to Quantify Shoulder Fatigue with Weighted Objects

Supplementary Material

A WICE CALCULATION

In this section, we provided a comprehensive and detailed calculation of the WICE model. Table 1 is the list of variables used in calculating WICE.

$$WICE \sim \mathcal{N}(\mu, \sigma)$$

$$WICE_{rest} = WICE \cdot \exp^{-0.04 \cdot \Delta t}$$

$$\mu = \frac{\Delta t \cdot \left(\frac{\tau_{shoulder} + C(\alpha_{shoulder})}{\tau_{max}} \cdot 100 \right)^{b} \cdot c}{a} \cdot 100$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (WICE_i - \mu)^2}$$

$$\tau_{max} = (227.338 + 0.525 \cdot \alpha_{elbow} - 0.296 \cdot \alpha_{shoulder}) \cdot G$$

$$G_{female} = 0.15, G_{male} = 0.28$$

$$\alpha_{elbow} = 180 - \arccos\left(\frac{\overrightarrow{Sh\ Eb} \cdot \overrightarrow{Eb\ Wr}}{\|\overrightarrow{Sh\ Eb}\| \cdot \|\overrightarrow{Eb\ Wr}\|} \right) \cdot \frac{180}{\pi}$$

$$\alpha_{shoulder} = \arccos\left(\frac{\overrightarrow{Sh\ Eb} \cdot [0, -1, 0]}{\|\overrightarrow{Sh\ Eb}\| \cdot \|[0, -1, 0]\|} \right) \cdot \frac{180}{\pi}$$

$$C_{female}(\alpha_{shoulder}) = \frac{0.0095 \cdot 1005}{1 + \exp\left(\frac{66.40 - \alpha_{shoulder}}{7.83}\right)} - \frac{\sin\left(\alpha_{shoulder} \cdot \frac{2\pi}{360}\right)}{0.11}$$
$$\{90^\circ < \alpha_{shoulder} < 180^\circ\}$$

$$C_{male}(\alpha_{shoulder}) = \frac{0.0095 \cdot 1230}{1 + \exp\left(\frac{66.40 - \alpha_{shoulder}}{7.83}\right)} - \frac{\sin\left(\alpha_{shoulder} \cdot \frac{2\pi}{360}\right)}{0.09}$$
$$\{90^\circ < \alpha_{shoulder} < 180^\circ\}$$

$$\tau_{shoulder,t} = \left\| \vec{r} \times \overrightarrow{force}_{motion,t} - \left(\vec{\tau}_{weight} + \vec{r} \times (M_{arm} \cdot \vec{g}) + \vec{I}_t \times \vec{\alpha}_t \right) \right\|$$

$$\vec{\tau}_{weight} = \vec{r}_o \times (M_{object} \cdot \vec{g})$$

$$\vec{g} = [0, -9.81, 0]$$

$$\vec{r}_o = \overrightarrow{Sh\ O}$$

$$\vec{r} = \overrightarrow{Sh\ CoM}$$

$$\overrightarrow{force}_{motion,t} = M_{arm} \cdot \vec{a}_t$$

$$\vec{a}_t = \frac{d^2 CoM}{dt^2}$$

$$\vec{I}_t = \left\| \vec{I}_{arm} \right\| \cdot \vec{U}_t$$

$$\vec{U}_t = \frac{\vec{r}_{t-1} \times \overrightarrow{CoM_{t-1}\ CoM_t}}{\left\| \vec{r}_{t-1} \times \overrightarrow{CoM_{t-1}\ CoM_t} \right\|}$$

$$\left\| \vec{I}_{arm} \right\| = \left\| \vec{I}_{UA} \right\| + \left\| \vec{I}_{FA} \right\| + \left\| \vec{I}_{H} \right\|$$

$$\vec{I} = \frac{\vec{r} \times (M_{segment} \cdot \vec{g})}{4\pi^2 f^2}$$

$$\vec{\alpha}_t = \frac{\vec{a}_t}{\|\vec{r}\|}$$

Below is the calculation of center of mass (CoM).

$$M_{UA} = 0.029 \cdot M_{body}$$

$$M_{FA} = 0.018 \cdot M_{body}$$

$$M_{H} = 0.008 \cdot M_{body}$$

$$M_{arm} = M_{UA} + M_{FA} + M_{H}$$

$$\vec{A} = 0.452 \cdot M_{UA} \cdot \overrightarrow{Sh\ Eb}$$

$$\vec{B} = 0.424 \cdot M_{FA} \cdot \overrightarrow{Eb\ Wr}$$

$$\vec{C} = 0.397 \cdot M_{H} \cdot \overrightarrow{Wr\ Ha}$$

$$\vec{D} = \vec{B} + \frac{H_m}{M_{FA} + M_{H}} \cdot \overrightarrow{BC}$$

$$\overrightarrow{CoM} = \vec{A} + \frac{M_{FA} + M_{H}}{M_{arm}} \cdot \overrightarrow{AD}$$

Table 1: The list of variables used in calculating WICE.

Variable	Unit	Description
$WICE$	%	The proposed exertion model when the user is active.
$WICE_{rest}$	%	The proposed exertion model when the user is rest.
μ	%	The mean exertion level per participant.
σ	%	The standard deviation of the exertion level per participant.
N	-	The number of participants.
t	-	The current timestamp.
Δt	s	The activate duration.
ET	s	The maximum time of the user could sustain such interaction before needing to rest the arm.
$C(\alpha_{shoulder})$	-	The correction term if the shoulder angle is above 90 degrees.
$\alpha_{shoulder}$	degree	The shoulder abduction angle.
α_{elbow}	degree	The elbow extension angle.
$\tau_{shoulder}$	Nm	The torque of the shoulder.
τ_{max}	Nm	The max torque of the shoulder. The values are different for male and female.
τ_{weight}	Nm	The weighted object torque acting on the shoulder.
$\vec{r}$	m	The distance from the shoulder joint to the center of mass.
$\vec{r}_o$	m	The distance from the shoulder joint to the center of the object.
$\overrightarrow{force}_{motion,t}$	N	The force acting at the center of mass at time t.
$\vec{g}$	N	The gravity acting on the arm, which is equal to the center of mass.
$\vec{a}_t$	m/s^2	The acceleration.
$\vec{I}_t$	$kg \cdot m^2$	The inertia of the arm at time t.
$\vec{U}_t$	-	The unit vector of the inertia of the arm.
$\vec{r}_{t-1}$	m	The distance from the shoulder joint to the center of mass at previous timestamp.
$\overrightarrow{CoM_{t-1}\ CoM_t}$	m	The center of mass from previous timestamp to the current timestamp.
$\vec{I}_{arm}$	$kg \cdot m^2$	The inertia of the arm.
$\vec{I}_{UA}$	$kg \cdot m^2$	The inertia of the upper arm.
$\vec{I}_{FA}$	$kg \cdot m^2$	The inertia of the forearm.
$\vec{I}_H$	$kg \cdot m^2$	The inertia of the hand.
$\vec{I}$	$kg \cdot m^2$	The inertia of an arm segment.
f	kg	The period of oscillation.
$\vec{\alpha}_t$	rad/s^2	The angular acceleration.
$\overrightarrow{CoM}$	N	The center of mass.
Sh	-	The location of the shoulder joint.
Eb	-	The location of the elbow joint.
Wr	-	The location of the wrist joint.
Ha	-	The location of the fingertip.
A	-	The center of mass of the upper arm segment.
B	-	The center of mass of the forearm segment.
C	-	The center of mass of the hand segment.
D	-	The center of mass of the forearm and hand segments.
O	-	The center of mass of the objects.
M_{UA}	kg	The mass of the upper arm segment.
M_{FA}	kg	The mass of the forearm segment.
M_H	kg	The mass of the hand segment.
M_{arm}	kg	The mass of the arm.
M_{body}	kg	The mass of the body.
M_{object}	kg	The mass of the object.
$M_{segment}$	s	The mass of an arm segment.

B BORG CR10 SCALES

Table 2: Borg CR10 scales with verbal anchoring.

Score	Definition	Note
0	Nothing At All	No arm fatigue
0.5	Very, Very Weak	Just noticeable
1	Very Weak	As taking a short walk
2	Weak	Light
3	Moderate	Somewhat but Not Hard to Go on
4	Somewhat Heavy	
5	Heavy	Tiring, Not Terribly Hard to Go on
6		
7	Very Strong	Strenuous. Really Push Hard to Go on
8		
9		
10	Extremely Strong	Extremely strenuous. Worst ever experienced